TRIANGULATED CATEGORIES OF RATIONAL EQUIVARIANT COHOMOLOGY THEORIES.

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1. Introduction.

This article is designed to provide an introduction to some examples of triangulated categories that arise in the study of G-equivariant cohomology theories for a compact Lie group G. We focus on cohomology theories whose values are rational vector spaces since one may often give explicit algebraic constructions of the triangulated category in that case.

As general references for equivariant cohomology theories see [3, 13, 14].

2. Examples of equivariant cohomology theories

Here are some examples of reduced equivariant cohomology theories on a based G-space X.

- **Borel cohomology theories:** $F^*(EG_+ \wedge_G X)$ for any non-equivariant cohomology theory $F^*(\cdot)$. [Here EG is the universal free G space, and EG_+ is the same space with a G-fixed basepoint adjoined].
- **Equivariant K-theory** $K_G^*(X)$: The theory is defined for unbased compact G-spaces Y by taking $K_G(Y)$ to be the Grothendieck group of equivariant vector bundles on Y. This defines $K_G^0(X) = ker(K_G(X) \longrightarrow K_G(*))$ in the usual way, and this is extended to all degrees by Bott periodicity. Note that $K_G^0 = K_G(*) = R(G)$, the complex representation ring, and $K_G^1 = 0$.
- **Equivariant Bordism** $MU_G^*(X)$**:** The stabilized form of bordism of G-manifolds with a complex structure on their stable normal bundle defined by tom Dieck. [15]

3. Definition of genuine cohomology theories

A naïve equivariant cohomology theory is a contravariant exact functor

$$F_G^* : \text{Based } G\text{-spaces} \longrightarrow \text{Graded abelian groups.}$$

If in addition $F_G^*(\cdot)$ is equipped with an extension to an $RO(G)$-graded theory in such a way that

$$F_G^{k+V}(S^V \wedge X) \cong F_G^k(X)$$

for any representation V (where S^V is the one point compactification of V), we say that $F_G^*(\cdot)$ is a 'genuine' equivariant cohomology theory. The Examples in Section 2 all have the stronger property that they are *complex stable* in the sense that

$$F_G^{k+|V|}(S^V \wedge X) \cong F_G^k(X)$$

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for every complex representation V. This is clear by the Serre spectral sequence for the Borel theories, it follows from Bott periodicity for equivariant K theory, and it is built into the definition for stabilized bordism.

One reason (beyond the existence of interesting examples) for considering genuine cohomology theories is that if $H \subseteq K$ there is an induction map $\mathrm{ind}_H^K : F_H^*(X) \longrightarrow F_K^*(X)$ in addition to the restriction map map $\mathrm{res}_H^K : F_K^*(X) \longrightarrow F_H^*(X)$ that exists for any naïve theory and is induced by the projection $G/H_+ \longrightarrow G/K_+$. Henceforth we drop the adjective 'genuine' since all cohomology theories will be genuine.

It is convenient to work in a context where equivariant cohomology theories are represented. Indeed, one may form the model category G-spectra [3], which can be thought of as a category of stable based G-spaces. Thus every based G-space gives rise to a suspension spectrum $\Sigma^\infty X$, and for every G-equivariant cohomology theory $F_G^*(\cdot)$ there is a G-spectrum F so that

$$F_G^*(X) = [\Sigma^\infty X, F]_G^*,$$

where $[A, B]^G$ means maps in the homotopy category of G-spectra. Henceforth we omit the notation Σ^∞ for the suspension spectrum functor.

As in the case of non-equivariant spectra, one may attempt to classify thick subcategories of finite G-spectra, but there are some additional complications. For instance, if X is a finite p-local G-spectrum the geometric fixed point spectrum X^H has a chromatic type $n_X(H)$. N.P.Strickland [18] has studied the functions n_X that can occur. For example, chromatic Smith theory shows that $n_X(H) \geq n_X(K) - 1$ if K is normal and of index p in H.

4. Ordinary equivariant cohomology and Mackey functors

The basic building blocks for G-spaces are the cells $(G/H \times D^n, G/H \times S^{n-1})$ for closed subgroups H and $n \geq 0$. Thus the relevant 0-spheres are G/H_+. Accordingly a cohomology theory $F_G^*(\cdot)$ satisfies the dimension axiom if $F_G^i(G/H_+) = 0$ for $i \neq 0$ and all closed subgroups H. A cohomology theory satisfying the dimension axiom is called an *ordinary* cohomology theory. Note also that

$$F_G^i(G/H_+) = [G/H_+, F]_G^i = [S^0, F]_H^i = \pi_{-i}^H(F)$$

so F represents an ordinary cohomology theory if and only if all its equivariant homotopy groups are concentrated in degree 0.

However the groups $F_G^i(G/H_+)$ for various subgroups H are related. First, define the stable orbit category $\mathcal{SO}$ to be the full subcategory of the homotopy category of G-spectra with objects G/H_+, it has morphisms $\mathcal{SO}(G/H_+, G/K_+) = \lim_{\to V} (S^V \wedge G/H_+, S^V \wedge G/H_+)^G$, where $(A, B)^G$ denotes homotopy classes of G-maps. We may then define an additive functor

$$\underline{\pi}_i^G(F) : \mathcal{SO} \longrightarrow Ab$$

by $\underline{\pi}_i^G(F)(G/H_+) = \pi_i^H(F)$. Quite generally, any additive functor $M : \mathcal{SO} \longrightarrow$ Ab is called a *Mackey functor*, and if we rewrite it by taking $M'(H) := M(G/H_+)$ then the way to think of a Mackey functor is that if $K \subseteq H$ then there is a restriction map $\mathrm{res}_K^H : M'(H) \longrightarrow M'(K)$ (induced by the projection $\pi : G/K \longrightarrow G/H$), a conjugation map $c_g : M'(H) \longrightarrow M'(H^g)$ (induced by right multiplication by g^{-1} as a map $G/H^g \longrightarrow G/H$), and an induction map $\mathrm{ind}_K^H : M'(K) \longrightarrow M'(H)$ (induced by a certain *stable* map $G/H \longrightarrow G/K$ (the dual of π if G is finite). These satisfy the Mackey induction restriction formula (or Feshbach's generalization if G is of positive dimension [2]). If G is finite there is a purely algebraic definition [1], which can be shown to be equivalent to this definition via topology.

Lemma 4.1. *Ordinary cohomology theories correspond bijectively to Mackey functors.*

Proof: We have seen that the zeroth homotopy group of an ordinary cohomology theory defines a Mackey functor, and conversely, given a Mackey functor M we may construct a cohomology theory $H^*_G(\cdot; M)$ by using cellular chain complexes, or alternatively construct the representing Eilenberg-MacLane G-spectrum HM directly by realising a resolution of M by free Mackey functors. $\square$

5. All cohomology is ordinary for finite groups.

It is an immediate consequence of Serre's calculation of the rational homotopy of spheres that every non-equivariant rational cohomology theory is ordinary. Here is a generalization to any finite group; a precursor for equivariant K-theory was the early result of Slominska [17]

Theorem 5.1. [12] *If G is a finite group then every rational cohomology theory $F^*_G(\cdot)$ is ordinary:*

$$F^k_G(X) \cong \prod_n H^{k+n}_G(X; \underline{\pi}^G_n(F)).$$

Proof: For the proof we define a related cohomology theory. Given any injective rational Mackey functor I we may define a cohomology theory $hI^*_G(\cdot)$ by

$$hI^n_G(X) = \mathrm{Hom}(\underline{\pi}^G_n(X), I),$$

There are two special facts about finite groups that let us proceed.

Lemma 5.2. *If G is finite every rational Mackey functor is injective.*

Proof: This is due to the fact that the rational Burnside ring splits as a product of copies of $\mathbb{Q}$ and Maschke's theorem. $\square$

For each n we may therefore choose the map $F \longrightarrow \Sigma^n h\underline{\pi}^G_n(F)$ corresponding to the identity map of $\underline{\pi}^G_n(F)$, and we may assemble these to give a map

$$F \xrightarrow{\simeq} \prod_n \Sigma^n h\underline{\pi}^G_n(F).$$

Lemma 5.3. *The spectrum hI is an Eilenberg-MacLane spectrum: $hI = HI$.*

Proof: Since $\underline{\pi}^G_0(G/H_+) = [\cdot, G/H_+]^G$ is the free functor, it is clear that hI has the correct homotopy groups in degree 0. We must calculate $hI^n_G(G/H_+)$ for each subgroup H, and show that it is zero if $n \neq 0$.

For this we need to examine the functor $\underline{\pi}^G_n(G/H_+)$, which is made up from the groups $\pi^K_n(G/H_+)$. The tom Dieck splitting theorem for the G-space X states

$$\pi^K_n(X) = \bigoplus_{(L)} \pi_n(EW_K(L)_+ \wedge_{W_K(L)} X^L)$$

where the sum is over K-conjugacy classes of subgroups L. Since we are working rationally, the homotopy may be replaced by homology, and since the groups concerned are finite, there is no higher homology; since $X = G/H_+$ is zero dimensional the result follows. □

It follows that the map ν induces an isomorphism of π_n^H for all n and H and is therefore an equivalence by the Whitehead theorem: the G-spectrum F splits as a product of Eilenberg-MacLane spectra

$$F \xrightarrow{\simeq} \prod_n \Sigma^n H \underline{\pi}_n^G(F).$$

The statement about cohomology theories follows. □

6. Algebraic models for categories of rational cohomology theories.

The idea is that for any compact Lie group G there is an abelian category $\mathcal{A}(G)$ modelling rational G-equivariant cohomology theories. On a practical level, we want to be able to *calculate* in this homotopy category, but if we understand the category completely we can also *construct* interesting new cohomology theories [8]. The idea is that objects of $\mathcal{A}(G)$ should be a rather small, and based on information easily accessible from the cohomology theories they represent.

Conjecture 6.1. *For a compact Lie group G there is an abelian category $\mathcal{A}(G)$ and a Quillen equivalence*

$$G\text{-spectra}/\mathbb{Q} \simeq dg\mathcal{A}(G)$$

such that

(1) *$\mathcal{A}(G)$ is abelian*
(2) $\mathrm{InjDim}(\mathcal{A}(G)) = \mathrm{rank}(G)$
(3) *The category consists of sheaves of modules over a space of closed subgroups of G; the object corresponding to a cohomology theory $E_G^*(\cdot)$ has fibre over H built from the Borel theory $E^*_{TW_G(H)}(ETW_G(H)_+ \wedge X^H)$. The additional structure is built from these Borel theories using their relation under localization and inflation.*
(4) *The model of $E_G^*(\cdot)$ is built from its values on spheres and a little extra structure.*

6.1. **Consequences of the conjecture.** Note immediately that if the conjecture holds we have an equivalence of homotopy categories

$$\mathrm{Ho}(G\text{-spectra}/\mathbb{Q}) \simeq D(\mathcal{A}(G))$$

as triangulated categories. This reduces to algebra the problem of classifying rational equivariant cohomology theories and the process of calculation with them. Furthermore, it provides a universal homology theory

$$\pi_*^{\mathcal{A}(G)} : G\text{-spectra} \longrightarrow \mathcal{A}(G)$$

and an Adams spectral sequence

$$\mathrm{Ext}^{*,*}_{\mathcal{A}(G)}(\pi_*^{\mathcal{A}(G)}(X), \pi_*^{\mathcal{A}(G)}(Y)) \Rightarrow [X,Y]^G_*$$

for calculation. Finally, because of the injective dimension of $\mathcal{A}(G)$, the Adams spectral sequence is only non-zero on s-line for $0 \leq s \leq r$, so the calculation is very accessible.

6.2. **Status of the conjecture.**

- G **finite.** The conjecture is true. From the result of Section 5 it is not hard to see
$$\mathcal{A}(G) = \prod_{(H)} \mathbb{Q}W_G(H)\text{-mod}.$$
- **The circle group** $G = T$**.** Again the theorem is true. Indeed, [5] constructs $\mathcal{A}(G)$ and shows that there is a triangulated equivalence of homotopy categories. Shipley [16] upgraded this to a Quillen equivalence. We describe the models for free T-spectra in Section 7 and the model for semifree spectra in Section 8.
- **The groups** $G = O(2), SO(3)$ **and their double covers.** In this case the equivalence of homotopy categories is proved in [6, 7].
- **The tori** $G = T^g$**.** The Adams spectral sequence exists [9], and in [10, 11] we show that the Quillen equivalence holds.

7. Free T-spaces.

We spend the rest of the article on the circle group $G = T$, and to simplify the discussion we consider actions with restricted isotropy. In this section we give a complete classification of rational cohomology theories on free T-spaces.

First note that an arbitrary space X is equivalent to a free T-space if and only if the map $ET_+ \wedge X \longrightarrow S^0 \wedge X = X$ is an equivalence. For homotopical work it is convenient to adopt this as the definition of a free space or spectrum.

Lemma 7.1. *Cohomology theories on free T-spaces are represented by free spectra.*

Proof: If X is free then $[X, F]^*_T \longleftarrow [X, F \wedge ET_+]^*_T$ is an isomorphism since maps from a free space into a non-equivariantly contractible space are null-homotopic. Hence $F^*_T(\cdot)$ is represented by the free T-spectrum $ET_+ \wedge F$. □

We may thus concentrate on classifying free T-spectra.

Lemma 7.2. *For any free X, the homotopy groups $\pi^T_*(X)$ are naturally a module over $\mathbb{Q}[c]$, where c is of degree -2. Furthermore, $\pi^T_*(X)$ is torsion in the sense that every element is annihilated by a power of c.*

Proof: Note that $[ET_+, ET_+]^T_*$ acts on $\pi^T_*(X) = \pi^T_*(X \wedge ET_+)$. Now calculate
$$[ET_+, ET_+]^T_* = [ET_+, S^0]^T_* = [BT_+, S^0]^T_* = H^*(BT_+) = \mathbb{Q}[c].$$
For the torsion statement, note that any element $x \in \pi^T_*(X)$ is supported on a finite subcomplex K, and $\pi^T_*(K)$ is bounded below. Since c is in degree -2, the statement follows. □

We are now equipped to state the classification.

Theorem 7.3. *Associating the module $\pi_*^T(F \wedge ET_+)$ to the cohomology theory $F_T^*(\cdot)$ gives a bijective correspondence*

$$\text{Cohomology theories on free } T\text{-spaces} \leftrightarrow \text{Torsion } \mathbb{Q}[c]\text{-modules}$$

Furthermore, for any free T-spectra X and F, there is a short exact sequence

$$0 \longrightarrow \mathrm{Ext}_{\mathbb{Q}[c]}(\pi_*^T(\Sigma X), \pi_*^T(F)) \longrightarrow [X, F]_*^T \longrightarrow \mathrm{Hom}_{\mathbb{Q}[c]}(\pi_*^T(X), \pi_*^T(F)) \longrightarrow 0.$$

We first need the Whitehead theorem.

Lemma 7.4. *If X and Y are free and $f : X \longrightarrow Y$ is a map inducing an isomorphism of π_*^T, then f is an equivalence.*

Proof: Change of groups and the Gysin sequence. □

Proof: The short exact sequence is an Adams spectral sequence. The method of proof is therefore standard. We need only note that there are realizable injectives F for which the short exact sequence exists, and that any free T-spectrum can be resolved by these.

To realize an injective, note that

$$\pi_*^T(ET_+) = \pi_*(\Sigma BT_+) = H_*(\Sigma BT_+) = \Sigma^{-1}\mathbb{Q}[c, c^{-1}]/\mathbb{Q}[c]$$

is c-divisible and hence injective. It is easy to show

$$[X, ET_+]_*^T \cong \mathrm{Hom}_{\mathbb{Q}[c]}(\pi_*^T(X), \pi_*^T(ET_+))$$

(both sides are cohomology theories in X so only need to check on $X = T_+$). This establishes the injective case.

Let us now show there are enough injectives of this form, and that they and the resulting resolutions are realizable. First, there are algebraically enough injectives of the form $\pi_*^T(ET_+)$. For simplicity assume that F is bounded below and of finite type. Hence we may construct an embedding $\pi_*^T(F) \longrightarrow \pi_*^T(\bigvee_i \Sigma^{n_i} ET_+)$ where the wedge is locally finite and hence equivalent to the product. We may then lift it to a map $F \longrightarrow \bigvee_i \Sigma^{n_i} ET_+ =: I$. Since $\mathbb{Q}$[c] is of injective dimension 1, the mapping cone J also has injective homotopy, and, as a matter of algebra, this is necessarily isomorphic to $\pi_*^T(\bigvee_j \Sigma^{n_j} ET_+)$ for suitable integers n_j. By the injective case of the short exact sequence we can construct a map from the cofibre to this wedge, and by 7.4 it is an equivalence. This gives a cofibre sequence $F \longrightarrow I \longrightarrow J$, realizing the injective resolution of $\pi_*^T(F)$, and where I and J are both wedges of suspensions of ET_+ (for which the theorem is known). Now apply $[X, \cdot]_*^T$ and obtain the exact sequence.

The classification of free T-spectra now follows easily. To construct enough spectra we realize a resolution of a torsion $\mathbb{Q}$[c]-module. To show that if $\pi_*^T(X) \cong \pi_*^T(Y)$ then $X \simeq Y$, we just lift the algebraic isomorphism to a map $X \longrightarrow Y$ and then apply 7.4 to deduce it is an equivalence. □

Corollary 7.5. *There is an equivalence of triangulated categories*

$$\text{Free } T\text{-spectra} \simeq D(\text{Isomorphism classes of torsion } \mathbb{Q}[c]\text{-modules})$$

where the derived category on the right is obtained from dg torsion $\mathbb{Q}$[c]-modules by inverting homology isomorphisms.

Proof: Use the Adams short exact sequence and the fact that c is in degree 2, together with a Toda bracket argument. $\square$

8. Semi-free T-spaces.

In this section we give a complete classification of rational cohomology theories on semi-free T-spaces (those whose isotropy groups are either 1 or T). The pattern of the argument is very similar to that in Section 7, so we will omit most of the proofs.

First note that an arbitrary space X is equivalent to a semi-free T-space if and only if the map $ET_+ \wedge X \longrightarrow E\mathcal{F}_+ \wedge X$ is an equivalence, where $E\mathcal{F}$ is the universal $\mathcal{F}$-space, where $\mathcal{F}$ is the set of finite subgroups. For homotopical work it is convenient to adopt this as the definition of a semi-free space or spectrum.

Lemma 8.1. *Cohomology theories on semi-free T-spaces are represented by semi-free spectra.* $\square$

Thus we turn to the study of semifree T-spectra.

Now, for any semi-free X we have a cofibre sequence

$$ET_+ \wedge X \longrightarrow X \longrightarrow \tilde{E}\mathcal{F} \wedge X,$$

where $\tilde{E}\mathcal{F} = \bigcup_{V^T=0} S^V$ is H-contractible for all finite H. We described the spectra $ET_+ \wedge X$ in Section 7, and it is easy to see that $\tilde{E}\mathcal{F} \wedge X \simeq \tilde{E}\mathcal{F} \wedge X^T$, so that $\tilde{E}\mathcal{F} \wedge X$ is determined by the graded rational vector space $\pi_*(X^T)$. It thus remains to describe how to splice these two pieces of information. For this we take the cue from the classical Localization theorem which states that if X is finite and semifree then

$$H^*(ET_+ \wedge_T X)[1/c] \cong H^*(ET_+ \wedge_T X^T)[1/c] \cong H^*(BT_+)[1/c] \otimes H^*(X^T) = \mathbb{Q}[c, c^{-1}] \otimes H^*(X^T).$$

Thus the Borel cohomology of X very nearly determines the cohomology of the fixed point space. Inspired by this we may define an appropriate category.

Definition 8.2. *The localization category $\mathcal{A}$ has objects $\beta : N \longrightarrow \mathbb{Q}[c, c^{-1}] \otimes V$, where N is a $\mathbb{Q}[c]$-module, and β is a $\mathbb{Q}[c]$-map which becomes an isomorphism when c is inverted. We call N the* nub, *V the* vertex *and β the* basing map. *The morphisms in $\mathcal{A}$ are given by commutative squares in which the map is the identity on $\mathbb{Q}[c, c^{-1}]$.*

The following lemma is an elementary exercise.

Lemma 8.3. *The category $\mathcal{A}$ is abelian and of injective dimension 1. In fact the objects $(I \longrightarrow 0)$ with I an injective torsion $\mathbb{Q}[c]$-module, and $(\mathbb{Q}[c, c^{-1}] \otimes V \xrightarrow{1} \mathbb{Q}[c, c^{-1}] \otimes V)$ together give enough injectives.* $\square$

The final three results are direct counterparts of results in Section 7.

Lemma 8.4. *For any semi-free X, the object*

$$\pi_*^{\mathcal{A}}(X) = \left(\pi_*^T(X \wedge DET_+) \longrightarrow \pi_*^T(X \wedge DET_+ \wedge \tilde{E}\mathfrak{F}) \cong \mathbb{Q}[c, c^{-1}] \otimes \pi_*(X^T)\right)$$

is an object of $\mathcal{A}$.

Proof: The cofibre of the map $X \wedge DET_+ \longrightarrow X \wedge DET_+ \wedge \tilde{E}\mathfrak{F}$ is $X \wedge DET_+ \wedge ET_+$; this is free and hence its homotopy is annihilated when c is inverted. □

We are now equipped to state the classification.

Theorem 8.5. *Associating the module $\pi_*^{\mathcal{A}}(F)$ to the cohomology theory $F_T^*(\cdot)$ gives a bijective correspondence*

Cohomology theories on semifree T-spaces $\leftrightarrow$ Isomorphism classes of objects of $\mathcal{A}$

Furthermore, for any semifree T-spectra X and F, there is a short exact sequence

$$0 \longrightarrow \mathrm{Ext}_{\mathcal{A}}(\pi_*^T(\Sigma X), \pi_*^T(F)) \longrightarrow [X, F]_*^T \longrightarrow \mathrm{Hom}_{\mathcal{A}}(\pi_*^T(X), \pi_*^T(F)) \longrightarrow 0. \quad \square$$

Corollary 8.6. *There is an equivalence of triangulated categories*

Semifree T-spectra $\simeq D(\mathcal{A})$

where the derived category on the right is obtained from dg objects of $\mathcal{A}$ by inverting homology isomorphisms. □

9. Some applications.

Here are some consequences which do not require much explanation to state.

- The Atiyah-Hirzebruch spectral sequence for $F_T^*(X)$ with X free collapses if and only if $\pi_*^T(F \wedge ET_+)$ is injective over $\mathbb{Q}$[c].
- (McClure) The Atiyah-Hirzebruch spectral sequence for $K_T^*(X)$ with X free always collapses.
- For an arbitary semifree space X, the K-theory $K_T^*(X)$ is determined by the map
$$H_*(ET_+ \wedge_T X^T) \longrightarrow H_*(ET_+ \wedge_T X).$$
- There are infinitely many non-isomorphic finite indecomposable semi-free spectra with $\pi_*^T(ET_+ \wedge X) \cong \pi_*^T(ET_+ \wedge (S^0 \vee S^2 \vee S^4))$ and $\pi_*(X^T) \cong \pi_*(S^0 \vee S^2 \vee S^4)$

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